Name: Math 130H Lab 2

The Poisson Random Variable

Once you have learned about what discrete random variables are in general, you can start to look at a few special random variables. In class we learned about one of these called a Binomial random variable where the variable X counts the number of successes out of a given number of runs of an experiment (trials). If you recognize that you are in a binomial random variable situation, then there are formulas to help you calculate some probabilities, the mean, the standard deviation and the variance, but they only work if you are sure you are in a binomial random variable situation. This lab will look at another special random variable, called the Poisson random variable.

Poisson Random Variable Situation

Let's start with an example first. Imagine that Lucy is a very popular girl and gets many phone calls on Saturday nights to see if she wants to go out. More specifically, let's say that Lucy gets an average of 10 phone calls on Saturday nights from 8:00pm to 9:00pm. Then if we want to know what the probability is that next Saturday night from 8:00pm to 9:00pm Lucy will get exactly 12 phone calls, then a Poisson random variable will do the job.

A few things to notice in this problem (to help you recognize that it is a Poisson problem)

1) An interval is given

   In this example, a time interval from 8:00pm to 9:00pm on Saturday nights is given. For all Poisson problems, there must be some distance or time interval in the problem.

2) The average amount of occurrences of something within the interval must be given

   In this example, we are given that the average amount of calls that Lucy gets on Saturday nights from 8:00pm to 9:00pm is 10 calls.

3) An occurrence of something is called a success

   A success in this problem means that Lucy received a call from 8:00pm to 9:00pm next Saturday night. If she gets 3 calls during that time, then there were 3 successes. If she gets no calls during that time, then there were no successes. Unlike the binomial distribution, there is no such thing as failure for a Poisson problem, just more and more successes.

4) The random variable X counts the number of successes within the interval

   Since this question is asking what the probability is that Lucy will get exactly 12 calls next Saturday night form 8:00pm to 9:00pm, the question is asking what the probability is that X=12. That is, it's asking for you to find $P(X = 12)$.

Your Turn: What are the possible values of X for a Poisson random variable? Can X=0? Is there a largest value that X can be? Remember that X is the number of occurrences of something in the interval.

5) There must be some notion of independence in a Poisson problem

   Just like the binomial random variable, there must be some notion of independence of successes.

Formulas for a Poisson Random Variable's Distribution

The derivation of the probability formulas for a Poisson random variable involve Calculus, so here I am just going to give them to you.

μ =the average number of successes in the interval will either be given in the problem, or information about it will be given so that you can calculate μ

$$P(X=x)=\frac{e^{-\mu}\mu^{x}}{x!} \qquad EV(X)=\mu \qquad \sigma^2=\mu \qquad \sigma=\sqrt{\mu}$$

Here, e is the number 2.718… which you have probably heard of in intermediate Algebra somewhere around the time you learned logs. Don't worry too much about the exact number because there is a button on your calculator for it.
The symbol ! is called factorial. What this means is that whatever number is in front of the factorial symbol, you write down all whole numbers from the number down to 1, then multiply. For example, 4! means $4\cdot3\cdot2\cdot1=24$. There is a button for this on your calculator also!

So to answer the question in the example above, we have

$$P(X=12)=\frac{e^{-10}10^{12}}{12!}=0.09478.$$

Your Turn: Here are 2 Poisson problems. Make sure to answer all parts and turn them in with this lab.

Ex 1: (Sec. 6.3 Ex 1, 2 from the book and more): A McDonald's manager knows from prior experience that cars arrive at the drive-through at an average rate of two cars per minute between the hours of 12:00pm and 1:00pm. Let X denote the number of cars that arrive between 12:00pm and 12:05pm. (Hint: Before you start this problem, you need to figure out what μ is. You might think that it's 2 cars per minute, but it's not because the time interval is 5 minutes long, not 1 minute long)

a) What is the probability that exactly 6 cars arrive between 12:00pm and 12:05pm?
b) What is the probability that fewer than 6 cars arrive between 12:00pm and 12:05pm?
c) What is the probability that at least 6 cars arrive between 12:00pm and 12:05pm?
d) Find the mean, variance and standard deviation of X
e) Explain what each of the probabilities you calculated in parts (a)-(c) mean.
f) Explain the meaning of the expected value you calculated in part (d)

Ex 2: The average number of cows you see on the side of the road on the trip from L.A. to Palm Springs is 4.7. On your next trip from L.A. to Palm Springs,

a) What is the probability that you will see exactly 4 cows?
b) What is the probability that you will see between 3 cows and 5 cows inclusive?
c) What is the probability that you will see at most 4 cows?
d) What is the probability that you will see more than 4 cows?
e) Find the mean, variance and standard deviation of the total number of cows you will see on the side of the road
f) Explain what each of the probabilities you calculated in parts (a)-(d) mean.
g) Explain the meaning of the expected value you calculated in part (e)
(In this problem, we have a distance interval instead of a time interval. The distance interval is the stretch of road from L.A. to Palm Springs)